PLATYPUS: a code for fusion and breakup in reactions induced by weakly-bound nuclei within a classical trajectory model with stochastic breakup

Alexis Diaz-Torres

Department of Nuclear Physics, Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia

Abstract

A self-contained Fortran-90 program based on a classical trajectory model with stochastic breakup is presented, which should be a powerful tool for quantifying complete and incomplete fusion, and breakup in reactions induced by weakly-bound two-body projectiles near the Coulomb barrier. The code calculates complete and incomplete fusion cross sections and their angular momentum distribution, as well as breakup observables (angle, kinetic energy and relative energy distributions).

PACS: 25.70.Jj, 25.70.Mn

Key words: Complete fusion cross section, Incomplete fusion cross section, Breakup cross section, Spin distribution, Kinetic energy distribution, Relative energy distribution, Angle distribution, Classical trajectory

PROGRAM SUMMARY/NEW VERSION PROGRAM SUMMARY

Manuscript Title: PLATYPUS: a code for fusion and breakup in reactions induced by weakly-bound nuclei within a classical trajectory model with stochastic breakup
Authors: Alexis Diaz-Torres
Program Title: PLATYPUS
Journal Reference:
Catalogue identifier:
Licensing provisions:
Programming language: Fortran-90
Computer: Any Unix/Linux workstation or PC
Operating system: Linux or Unix
RAM: 9.4 MB
Keywords: Complete fusion cross section, Incomplete fusion cross section, Breakup

cross section, Spin distribution, Kinetic energy distribution, Relative energy distribution, Angle distribution, Classical trajectory

PACS: 25.70.Jj, 25.70.Mn

Classification:

External routines/libraries:

Several source routines from Numerical Recipies, and the Mersenne Twister random number generator package are included to enable independent compilation.

Nature of problem:

The program calculates complete and incomplete fusion cross sections and their spin distribution, as well as breakup observables (e.g. the angle, kinetic energy, and relative energy distributions of the fragments) in reactions induced by weakly-bound two-body nuclei near the Coulomb barrier.

Solution method:

A classical trajectory model with stochastic breakup is used to calculate all the observables. See Ref. [1] for further details.

Restrictions:

The program is suited for weakly-bound two-body projectiles. The initial orientation of the segment joining the two breakup fragments is considered to be isotropic.

Running time:

About one hour for input provided, using a PC with 1.5 GHz processor.

References:

[1] A. Diaz-Torres et al., Phys. Rev. Lett. **98**(2007) 152701.

1 Introduction

Recent developments of radioactive isotope accelerators enable the investigation of fusion reactions that form heavy elements in the cosmos. These involve reactions of nuclei far from stability. The most exotic of these are often very weakly bound. Breakup of weakly bound nuclei is an important process in their reactions with other nuclei. A major consequence of breakup is that not all the resulting breakup fragments might be captured by the target, termed incomplete fusion (ICF); capture of the entire projectile by the target is called complete fusion (CF). Such ICF processes can dramatically change the nature of the reaction products, as has been investigated in detail for the stable weakly-bound nuclei ^{9}Be and ^{6,7}Li [1]. There, at energies above the fusion barrier, CF yields were found to be only $\sim 2/3$ of those expected, the remaining $1/3$ being in ICF products. Events where the projectile breaks up and none of the fragments are captured provide an important diagnostic of the reaction dynamics. This we call no-capture breakup (NCBU), which is expected to be predominant at energies below the fusion barrier. The modelling of all these reaction processes within the same theoretical framework is an outstanding problem. The continuum-discretised-coupled-channels (CDCC) quantum mechanical model can make reliable predictions of the NCBU process, but cannot

distinguish between ICF and CF processes unambiguously [2]. An alternative for solving this problem is the development of classical dynamical approaches based on the concept of classical trajectories, which are being successfully applied to describe atomic many-body collisions, including ionization [4], a process in some ways analogous to breakup.

In this paper, we present a program of the classical trajectory model with stochastic breakup, recently published by the author et al. in Ref. [3]. Since the code could be very useful for many researchers involved in near-barrier experiments of fusion and breakup with weakly-bound/ radioactive nuclei, it will be beneficial make the program accessible to everyone. In Section 2, a description of the model is given to make the paper self-contained, whilst in Section 3 the program and the input file are explained. The code is illustrated in section 4 with the reaction of a pseudo-^{8}Be projectile P (assuming a weakly-bound state of two α-particles) with a ^{208}Pb target. For this test case, the good agreement of the classical model calculations for the NCBU process with those of the CDCC quantum mechanical model shows the reliability of the classical approach (see Ref. [3] for details).

2 Model

2.1 Projectile-target interaction

The weakly bound (two-body) projectile P, with incident energy E_0 and orbital angular momentum L_0, is incident on the target T, initially at rest in the laboratory frame. Prior to breakup the projectile follows a classical orbit, which is characterised by a P-T distance of closest approach $R_{min}(E_0, L_0)$. These orbits of the bound projectile are calculated by numerical solution of the Newtonian equations of motion in the presence of the Coulomb and nuclear potentials between P and T. These generate a fusion barrier for head-on ($L_0 = 0$) collisions having a height V_B^{PT} at an internuclear distance R_B^{PT}.

2.2 Encoding of breakup

The Coulomb and nuclear two-body interactions that cause breakup are empirically encoded in a density of (local breakup) probability $\mathcal{P}_{BU}^{L}(R)$, which is a function only of the projectile-target separation R. It is defined such that $\mathcal{P}_{BU}^{L}(R)dR$ is the probability of breakup in the interval R to $R + dR$. A key feature that must be emphasised is that for a given projectile-target combination, both experimental data [5] and CDCC calculations indicate that the

integral of this breakup probability along a given classical orbit is an exponential function of its distance of closest approach, $R_{min}(E_0, L_0)$. Explicitly (see Appendix A),

$$P_{BU}(R_{min}) = 2 \int_{R_{min}}^{\infty} \mathcal{P}_{BU}^{L}(R)dR = A \exp(-\alpha R_{min}). \tag{1}$$

It follows uniquely from this equation that the local breakup function has the same exponential form,

$$\mathcal{P}_{BU}^{L}(R) \propto \exp(-\alpha R). \tag{2}$$

2.3 Initial conditions of breakup events

The code then proceeds with a Monte Carlo approach to sample the initial conditions of breakup. For each L_0 (chosen to be an integer number of $\hbar$) a sample of N incident projectiles is taken. The position of breakup on this orbit is determined by sampling a breakup radius R_{BU} in the interval $[R_{min}(E_0, L_0), \infty]$ with the weighting $\mathcal{P}_{BU}^{L}(R)$. Although dependent on the constants A and α, the exponential weighting of $\mathcal{P}_{BU}^{L}(R)$ will clearly place most R_{BU} in the vicinity of R_{min}. If the chosen L_0 is less than the critical partial wave for projectile fusion, L_{cr}, then the associated trajectory would normally lead to CF, i.e. $R_{min} \leq R_B^{PT}$. For these L_0 we set $R_{min} = R_B^{PT}$, when sampling R_{BU}, and all breakup events are confined to the incoming branch of the projectile trajectory. For $L_0 > L_{cr}$ breakup can take place on both the entrance and the exit branches of the classical orbit, which are sampled equally. Having chosen the position with the projectile center of mass at breakup, it is instantaneously broken up into fragments F1 and F2.

Following breakup, the two fragments F1 and F2 interact with T, and with each other, through real central two-body potentials having Coulomb barriers V_B^{ij} at separations R_B^{ij}, $i, j = 1, 2, T,\ i \neq j$. The instantaneous dynamical variables of the excited projectile at breakup, namely its total internal energy ε_{12}, its angular momentum $\vec{\ell}_{12}$ and the separation of the fragments $\vec{d}_{12}$ are all Monte Carlo sampled. The initial separations d_{12} between the fragments are Gaussian distributed in their classically allowed region. This mimics the radial probability distribution of the projectile ground-state (g.s.) wave function. For high ℓ_{12}, when there is no barrier between F1 and F2, d_{12} is taken as their external turning point. In the test calculation described below, the orientation of $\vec{d}_{12}$ is chosen randomly over the 4π solid angle, and the orientation of $\vec{\ell}_{12}$ is chosen randomly from all directions orthogonal to $\vec{d}_{12}$. Other distributions could be taken in cases where the two breakup fragments are not identical. ℓ_{12} is sampled uniformly in the interval $[0, \ell_{max}]$. For ε_{12} two sampling functions

were tested ranging from the energy of the top of the barrier, V_B^{12}, to a chosen maximum ε_{max}. Using uniform weighting or an exponentially decreasing weighting yielded very similar outcomes. The convergence of the observables was faster for the latter case, which was therefore used. Both ℓ_{max} and ε_{max} were increased until convergence of the observables occured, as in the CDCC calculations.

2.4 Trajectories of breakup fragments and the target

Having fixed the positions and dynamical variables of the excited projectile fragments at the moment of breakup, the instantaneous velocities of the particles F1, F2 and T are determined by conservation of energy, linear momentum and angular momentum in the overall center of mass frame of the projectile and target system (see Appendix B for details). These breakup initial conditions were transformed to the laboratory frame where the equations of motion are solved. The calculated trajectories of F1, F2 and T determine the number of ICF, CF and NCBU events, fragment Fj being assumed to be captured if the classical trajectories take it within the fragment-target barrier radius R_B^{jT}.

2.5 Probabilities and cross sections

From the N breakup events sampled for each projectile angular momentum L_0, the numbers of events N_i in which $i = 0$ (NCBU), 1 (ICF), or 2 (CF) fragments are captured determine the relative yields $\widetilde{P}_i = N_i/N$ of these three reaction processes after breakup, with $\widetilde{P}_0 + \widetilde{P}_1 + \widetilde{P}_2 = 1$. The absolute probabilities $P_i(E_0, L_0)$ of these processes are expressed in terms of the relative yields and the integrated breakup probability over the whole trajectory $P_{BU}(R_{min})$:

$$
\begin{aligned}
P_0(E_0, L_0) &= P_{BU}(R_{min})\,\widetilde{P}_0, && (3)\\
P_1(E_0, L_0) &= P_{BU}(R_{min})\,\widetilde{P}_1, && (4)\\
P_2(E_0, L_0) &= [1 - P_{BU}(R_{min})]\,H(L_{cr} - L_0) \\
&\quad + P_{BU}(R_{min})\,\widetilde{P}_2, && (5)
\end{aligned}
$$

where $H(x)$ is the Heaviside step function. The cross sections are calculated using

$$\sigma_i(E_0) = \pi\lambda^2 \sum_{L_0} (2L_0 + 1) P_i(E_0, L_0), \tag{6}$$

where $\lambda^2 = \hbar^2/[2m_P E_0]$ and m_P is the projectile mass.

The other observables, such as the angle, kinetic energy and relative energy distributions of the fragments from NCBU events, are calculated by tracking their trajectories to a large distance from the target, which was 200 fm in the calculations given below.

3 Computer program and input file

3.1 Structure of the code

The code has a main program and eight modules. The main program *breakup3D* directs the input to be read, the problem to be solved, and details of the calculation to be written in output files. The modules are *global data, potentials, mt19937, nrutil, initial conditions, fusion, Angular momentum distribution* and *INPUT VALUES*.

The main program *breakup3D* calls the module *INPUT VALUES* first, in which the subroutine *input data* reads the input file described below. For a given partial wave between the projectile and the target (IMPACTMIN up to IMPACTMAX), the subroutines *projectile trajectory* and *trajectory arrays* of this module calculate the orbit of the bound projectile and store it for interpolations in the module *initial conditions*. Thereafter, ISEEDMAX breakup events with sampled initial conditions are calculated for every partial wave. The initial conditions for the propagation in time of the three bodies are fixed by the subroutine *initial values* of the module *initial conditions*, whilst the classical trajectory of the breakup fragments and the target are calculated by the driver-subroutine *ODEINT* of the module *nrutil*. During the propagation, the possible capture of the breakup fragments by the target is determined by the subroutine *fusion events* in the module *fusion*. Finally, the spin distribution and cross sections for CF and ICF, and for the breakup process are calculated by the subroutine *spin distribution* in the module *Angular momentum distribution*. Here, other observables such as the angle, kinetic energy and relative energy distributions of the fragments from NCBU events are also computed. Depending on the value of the output control variables (FILE1, FILE2, FILE3 and FILE4), details of the calculation can be written into output files.

Module *global data*. This defines global variables used in different modules.

Module *potentials*. Here the nuclear and Coulomb interactions between the participants of the reaction (projectile fragments and the target) are defined.

Module *mt19937*. It contains the Mersenne Twister random number generator, written by Makoto Matsumoto and Takuji Nishimura [6].

Module *nrutil*. In this module several subroutines from Numerical Recipies are included, which are mainly related to integrating the classical equations of motion. *ODEINT* is the integrator driver that calls the subroutine *RKQC* which is a fouth-order Runge-Kutta integrator ensuring accuracy and adjusted stepsize. The forces are defined in the subroutine *DERIVS*. The module also contains subroutines (*printout1* and *printout2*) for writing into output files the trajectory of the nuclei along with the value of the integrals of motion. The latter is crucial for checking the accuracy of the integration of the classical equations of motion.

Module *initial conditions*. This module defines the initial conditions for the propagation in time of the three bodies, following the breakup of the two-body projectile. This task is performed by the subroutine *initial values*, which is guided by the conservation of energy, linear momentum and angular momentum in the overall center-of-mass frame of the projectile and target system. This is transformed to the laboratory reference frame (with Galilean kinematical relations) where the equations of motion are solved using a system of spherical coordinates. This module calls the module *mt19937* in the sampling of (i) the initial excitation energy of the projectile, (ii) the initial relative angular momentum between the projectile fragments, (iii) the initial separation between the fragments, (iv) the breakup radius, (v) the initial orientation of the segment joining the two fragments of the projectile, and (vi) the initial direction of the radial velocity along that segment. If the output control variables are activated, the initial conditions will be written into output files.

Module *fusion*. In this module the subroutine *fusion events* analyses the possible capture of any of the breakup fragments by the target. Here, the relative energy and angular momentum between the nuclei during the propagation are calculated by the subroutines *relative energy* and *relative spin*.

Module *Angular momentum distribution*. Knowing the statistics of fusion and NCBU events after a large number of sampling (1000 per partial wave in the example given below), the subroutine *spin distribution* calculates observables related to CF and ICF processes (spin distribution and cross sections). The angle, kinetic energy and relative energy distributions of the NCBU events and the breakup cross section are also computed here. This subroutine extensively calls the subroutine *LOCATE* of the module *nrutil*. All observables are written into output files.

Module *INPUT VALUES*. Here the input file is read by the subroutine *input data*. It also calculates the s-barrier features (radius and height) between (i) the projectile and the target, (ii) the projectile fragments and the target, and (iii) the two fragments of the projectile. The module also contains the subroutines *projectile trajectory* and *trajectory arrays* mentioned above.

3.2 *Input file*

Fig. 1 shows the input file of the program. At the bottom of the figure, the namelist of the input variables appearing in the code are shown. The lines related to the potentials (lines 12–19) are self-explanatory, whilst some variables in previous lines have already been mentioned in the description of the program. We will here describe lines 1–10 only.

The integer variables of the first line allow the user to write details of the dynamical calculations into output files, when they are equal to one. FILE1 and FILE2 open output files to write the trajectory of the breakup fragments and the initial projectile-target, respectively. The file opened by FILE3 contains details of breakup events, and FILE4 is for plotting the trajectory of the breakup fragments. Line 2 defines the window of orbital angular momentum of the incident projectile (IMPACTMIN, IMPACTMAX) in units of $\hbar$, and the number of breakup events per partial wave (ISEEDMAX). PARTICULAR IMPACT in line 3 is to select a partial wave, whose associated dynamical calculations are written into the file opened by FILE3. Line 4 refers to the incident energy of the projectile in MeV (E0). Lines 5–6 define the range of initial excitation energy (EXCMIN, EXCMAX) in MeV and relative angular momentum (L12MIN, L12MAX) in units of $\hbar$ for the breakup fragments. In line 7, TYPE EXC controls the sampling function for excitation where for TYPE EXC=0 the weighting is uniform, and for TYPE EXC=1 it is exponentially decreasing with a coefficient alfaexc. Line 8 defines the centroid (d012) and width (sig012) in fm of the Gaussian that describes the radial probability distribution of projectile ground-state wave function. It is used to sample the initial separation between the breakup fragments. The breakup function [see expression (1) in Section 2] is given in line 9 by the parameters alpha and beta, being beta=ln(A). Line 10 defines a maximal projectile-target separation for sampling the breakup radius.

4 Test run

Using the input file of Fig. 1, Figs. 2, 3 and 4 show output files for angular momentum distribution in ICF, CF and NCBU processes, respectively. The angular momenta are in the first column, whilst in the second and third columns are partial probabilities and cross sections. The total cross section appears at the end of the file. In Fig. 2, the ICF spin distribution refers to either the angular momentum brought by the captured fragment into the target or the orbital angular momentum of the incident projectile. The former has been employed very recently to understand isomer ratio measurements [7]. Of course, both distributions are the same for CF. Since the component of the CF spin distri-

```
0 0 0 0
000 045 01000
000
050.00
06.00 04.00
01.05 00.00
1 00.922
01.80 01.00
0.9220 9.7300
050.00
#######POTENTIALS################
208.0 082.0
008.0 004.0
004.0 002.0
004.0 002.0
-120.9030 1.3900 0.7552 1.2000
-062.0000 1.3900 0.6200 1.2000
-062.0000 1.3900 0.6200 1.2000
-016.6964 1.2000 0.6200 1.2000
##################################
Line 1: FILE1, FILE2, FILE3, FILE4
Line 2: IMPACTMIN, IMPACTMAX, ISEEDMAX
Line 3: PARTICULAR_IMPACT
Line 4: E0
Line 5: EXCMAX, L12MAX
Line 6: EXCMIN, L12MIN
Line 7: TYPE_EXC, alfaexc (only important when TYPE_EXC=1)
Line 8: d012,sig012
Line 9: alpha, beta
Line 10: RBU_max
Line 12: Mass and charge of the target (AT,ZT)
Line 13: Mass and charge of the projectile (AP,ZP)
Line 14: Mass and charge of the fragment1 (AP1,ZP1)
Line 15: Mass and charge of the fragment2 (AP2,ZP2)
Line 16: Target-Projectile potential (V0TP,rr0TP,a0TP)
      and Coulomb radius (rrc0TP)
Line 17: Target-Fragment1 potential (V01,rr01,a01) and
      and Coulomb radius (rrc01)
Line 18: Target-Fragment2 potential (V02,rr02,a02) and
      and Coulomb radius (rrc02)
Line 19: Fragment1-Fragment2 potential (V012,rr012,a012) and
      and Coulomb radius (rrc012)
```

Fig. 1. Input file for PLATYPUS code (PLATYPUSinp). See text for further details.

bution associated with the first term of expression (5) (fusion of the bound projectile) is clearly equal to $\pi\lambda^2(2L_0+1)[1-P_{BU}(R_{min})]$, in Fig. 3 we present only the nontrivial component related to the capture of all the projectile fragments after breakup. The code also calculates CF cross sections related to the inert projectile and to the mentioned two components of this process when the projectile can be dissociated, as shown at the end of Fig. 3. As expected, the NCBU process in Fig. 4 reveals a broader angular momentum distribution than the CF and ICF processes.

A Appendix

REGARDING ANGULAR MOMENTUM BROUGHT
BY FRAGMENTS INTO TARGET

L(hbar) P_L SIGMA2_L (mb)

******SMOOTH DISTRIBUTIONS*****

0.00	.725E-02	0.8705E-01
1.00	.857E-01	0.1232E+01
2.00	.221E+00	0.4141E+01
3.00	.423E+00	0.9467E+01
4.00	.635E+00	0.1611E+02
5.00	.756E+00	0.2147E+02
6.00	.942E+00	0.3059E+02
7.00	.102E+01	0.3729E+02
8.00	.983E+00	0.4120E+02
9.00	.762E+00	0.3420E+02
10.00	.454E+00	0.2106E+02
11.00	.274E+00	0.1264E+02
12.00	.177E+00	0.7912E+01
13.00	.100E+00	0.4533E+01
14.00	.357E-01	0.1594E+01
15.00	.178E-01	0.8071E+00
16.00	.396E-02	0.1870E+00
17.00	.132E-02	0.6663E-01

****

TOTAL SIGMA_ICF =0.2446E+03 mb

REGARDING PROJECTILE ANGULAR MOMENTUM

0	0.8439E-01	0.1376E+00
1	0.2696E+00	0.1319E+01
2	0.2657E+00	0.2166E+01
3	0.3019E+00	0.3446E+01
4	0.2670E+00	0.3917E+01
5	0.3039E+00	0.5450E+01
6	0.3072E+00	0.6511E+01
7	0.3395E+00	0.8302E+01
8	0.3474E+00	0.9629E+01
9	0.4134E+00	0.1280E+02
10	0.4127E+00	0.1413E+02
11	0.4200E+00	0.1575E+02
12	0.4740E+00	0.1932E+02
13	0.4813E+00	0.2118E+02
14	0.5070E+00	0.2397E+02
15	0.4582E+00	0.2316E+02
16	0.3896E+00	0.2096E+02
17	0.3323E+00	0.1896E+02
18	0.2657E+00	0.1603E+02
19	0.1160E+00	0.7377E+01
20	0.5708E-01	0.3815E+01
21	0.4295E-01	0.3011E+01
22	0.1970E-01	0.1445E+01
23	0.1378E-01	0.1056E+01
24	0.5048E-02	0.4033E+00
25	0.2500E-02	0.2078E+00
26	0.6901E-03	0.5963E-01
27	0.4582E-03	0.4108E-01
28	0.1903E-03	0.1769E-01
29	0.1582E-03	0.1521E-01

SIGMA2_ICF=0.2446E+03 mb

Fig. 2. ICF angular momentum distribution (ICF SPIN DISTRIBUTION). See text for further details.

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REGARDING ANGULAR MOMENTUM BROUGHT
BY FRAGMENTS INTO TARGET

L(hbar)  P_L  SIGMA2_L (mb)


******SMOOTH DISTRIBUTIONS*****

  0.00  .572E+00  0.9318E+00
  1.00  .385E+00  0.1883E+01
  2.00  .388E+00  0.3160E+01
  3.00  .354E+00  0.4040E+01
  4.00  .384E+00  0.5630E+01
  5.00  .353E+00  0.6325E+01
  6.00  .344E+00  0.7293E+01
  7.00  .312E+00  0.7641E+01
  8.00  .299E+00  0.8295E+01
  9.00  .230E+00  0.7127E+01
 10.00  .223E+00  0.7651E+01
 11.00  .208E+00  0.7811E+01
 12.00  .142E+00  0.5804E+01
 13.00  .101E+00  0.4440E+01
 14.00  .475E-01  0.2244E+01
 15.00  .218E-01  0.1099E+01
 16.00  .165E-01  0.8867E+00
 17.00  .857E-02  0.4890E+00
 18.00  .593E-02  0.3579E+00
 19.00  .198E-02  0.1257E+00

*********************************

TOTAL SIGMA_CF =0.8323E+02 mb

SIGMA_CF(INERT PROJ)=0.6521E+03 mb
SIGMA2_CF=0.8323E+02 mb
SIGMA_CF(NO BU)=0.2222E+03 mb
SIGMA_CF(TOT)=0.3054E+03 mb
```

Fig. 3. CF angular momentum distribution (CF SPIN DISTRIBUTION). See text for further details.

Let us define two probabilities: (i) the probability of breakup between R and $R+dR$, $\rho(R)dR$ [being $\rho(R)$ a density of probability], and (ii) the probability the weakly-bound projectile has survived from ∞ to R, $S(R)$. The survival probability at $R+dR$, $S(R+dR)$, can be written as follows

$$S(R+dR) = S(R)\,[1-\rho(R)dR]. \tag{7}$$

Expression (7) suggests the following differential equation for the survival probability $S(R)$,

$$\frac{dS(R)}{dR} = -S(R)\,\rho(R), \tag{8}$$

whose solution is $[S(\infty)=1]$:

$$S(R) = \exp(-\int_{\infty}^{R} \rho(R)dR). \tag{9}$$

REGARDING PROJECTILE ANGULAR MOMENTUM

L(hbar)	PBU_L	SIGMA_L
0	0.3296E-02	0.5374E-02
1	0.4615E-02	0.2257E-01
2	0.5934E-02	0.4836E-01
3	0.3296E-02	0.3762E-01
4	0.8571E-02	0.1257E+00
5	0.2637E-02	0.4729E-01
6	0.7911E-02	0.1677E+00
7	0.7252E-02	0.1773E+00
8	0.1253E-01	0.3471E+00
9	0.1582E-01	0.4901E+00
10	0.2307E-01	0.7899E+00
11	0.3099E-01	0.1162E+01
12	0.4285E-01	0.1746E+01
13	0.7714E-01	0.3395E+01
14	0.1048E+00	0.4956E+01
15	0.1793E+00	0.9062E+01
16	0.2532E+00	0.1362E+02
17	0.3184E+00	0.1817E+02
18	0.3877E+00	0.2338E+02
19	0.5327E+00	0.3387E+02
20	0.4254E+00	0.2843E+02
21	0.3318E+00	0.2326E+02
22	0.2740E+00	0.2010E+02
23	0.2287E+00	0.1752E+02
24	0.1930E+00	0.1542E+02
25	0.1590E+00	0.1322E+02
26	0.1337E+00	0.1156E+02
27	0.1115E+00	0.9993E+01
28	0.9050E-01	0.8410E+01
29	0.7569E-01	0.7280E+01
30	0.6349E-01	0.6314E+01
31	0.5241E-01	0.5383E+01
32	0.4346E-01	0.4606E+01
33	0.3628E-01	0.3963E+01
34	0.2994E-01	0.3368E+01
35	0.2495E-01	0.2888E+01
36	0.2062E-01	0.2453E+01
37	0.1704E-01	0.2083E+01
38	0.1407E-01	0.1766E+01
39	0.1157E-01	0.1490E+01
40	0.9551E-02	0.1261E+01
41	0.7880E-02	0.1066E+01
42	0.6482E-02	0.8982E+00
43	0.5350E-02	0.7587E+00
44	0.4398E-02	0.6381E+00
45	0.3553E-02	0.5271E+00

SIGMA_BU=0.3063E+03 mb

Fig. 4. NCBU angular momentum distribution (BU SPIN DISTRIBUTION). See text for further details.

From (9), the breakup probability at R, $B(R) = 1 - S(R)$. If $\int_\infty^R \rho(R)dR \ll 1$, $B(R)$ can be written as

$$B(R) \approx \int\limits_\infty^R \rho(R)dR. \tag{10}$$

From (10), identifying $\rho(R)$ with $\mathcal{P}_{BU}^L(R)$, we obtain expression (1) for the breakup probability integrated along a given classical orbit.

B Appendix

The integrals of motion in the overall center-of-mass (CM) system are the total energy $E_{tot} = \frac{m_T}{(m_T+m_P)} E_0$, the total linear momentum $\vec{P}_{tot} = \vec{0}$, and the total angular momentum $\vec{L}_{tot} = m_P b_0(\vec{v} - \vec{V}_{CM})$ that is orthogonal to the initial reaction plane. m_T, m_P, b_0, $\vec{v}$, and $\vec{V}_{CM}$ are the mass of the target and projectile, the impact parameter between the projectile and the target, the velocity of the incident projectile in the laboratory system and the CM velocity, respectively.

Just after breakup, the two-body projectile is excited to a definite state (ε_{12}, $\vec{\ell}_{12}$ and $\vec{d}_{12}$), as explained in Section 2. The relative vector between P and T ($\vec{R}_{PT}$) is also known. Thus, the separation between the three bodies is known. The modulus of the velocity between P and T ($V_{PT} = P_{PT}/\mu_{PT}$) results from the total energy conservation

$$E_{tot} = \varepsilon_{12} + U_{1T}(r_{1T}) + U_{2T}(r_{2T}) + P_{PT}^2/2\mu_{PT}, \tag{11}$$

where U is the interaction potential between the target and the breakup fragments.

The total linear momentum $\vec{P}_{tot} = \vec{p}_T + \vec{p}_1 + \vec{p}_2 = \vec{p}_T + \vec{p}_{P^*}$, where $\vec{p}_{P^*}$ is the momentum of the center of mass of excited P relative to the overall CM. We need the velocities of P and T relative to the overall CM ($\vec{\tilde{v}}_P$ and $\vec{\tilde{v}}_T$) to complete the initial conditions for subsequent propagation in time of the three bodies. These velocities are related to each other by the expressions

$$\vec{\tilde{v}}_T = -\frac{m_P}{m_T}\vec{\tilde{v}}_P, \tag{12}$$

$$\vec{V}_{PT} = \vec{\tilde{v}}_P - \vec{\tilde{v}}_T, \tag{13}$$

where the magnitude of $\vec{V}_{PT}$ is known through expression (11). To know the direction of this velocity the conservation of total angular momentum is applied.

The total angular momentum $\vec{L}_{tot} = \vec{\ell}_{12} + \vec{L}_{PT}$, so the angular momentum ($\vec{L}_{PT}$) associated with relative motion of P and T about CM is known. This vector can be written as

$$\vec{L}_{PT} = m_P \, \vec{R}_{PT} \times \vec{\tilde{v}}_P. \tag{14}$$

We now write $\vec{\tilde{v}}_P$ in terms of radial and transverse components as follows:

$$\vec{\tilde{v}}_P = \tilde{v}_P^{(r)} \, \vec{r} + \tilde{v}_P^{(q)} \, \vec{q}, \tag{15}$$

where $\vec{r} = \vec{R}_{PT}/R_{PT}$ and $\vec{q} = \vec{n} \times \vec{r}$, being $\vec{n} = \vec{L}_{PT}/L_{PT}$. The transverse component $\tilde{v}_P^{(q)} = L_{PT}/(m_P R_{PT})$, and for the target $\tilde{v}_T^{(q)} = -L_{PT}/(m_T R_{PT})$. The radial component is obtained using expressions (12)-(13) and knowing the transverse component:

$$\tilde{v}_P^{(r)} = \pm \Big\{ V_{PT}^2 - \Big[\tilde{v}_P^{(q)} \, (1 + \frac{m_P}{m_T}) \Big]^2 \Big\}^{1/2} / (1 + \frac{m_P}{m_T}). \tag{16}$$

Both positive and negative roots are consistent with the conservation of the integrals of motion. Hence, both roots are uniformly sampled. Finally, the position and velocity vectors of the projectile fragments and the target are transformed to the laboratory system using Galilean transformations.

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